Much more than counting

Supporting mathematics development between birth and five years

Cally Kühne, Shelley O'Carroll, Brigid Comrie
and Rebecca Hickman
with contributions from Karen van der Merwe

SDU
Schools Development Unit
School of Education

wordworks
transforming lives through literacy

University of Cape Town
Schools Development Unit
Tel: 021 650 3368
email: cally.kuhne@uct.ac.za
www.sdu.uct.ac.za

Wordworks
Tel: 021 788 9233
email: info@wordworks.org.za
www.wordworks.org.za

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Introduction

IN ORDER TO REDRESS the effects of poverty and reverse the stubborn trend of educational underachievement in South Africa, the right foundations for learning must be laid in the period before children begin formal schooling. Historically, the purpose and importance of mathematics learning in the early years has not been properly understood or valued in South Africa. This is in part because there is not a general understanding of how mathematical ideas and skills are developed between birth and five years, nor of how best to approach the teaching and learning of key concepts and competencies during this period.

This report seeks to fill that gap, by providing both a conceptual overview and practical framework for early mathematics development. It is aimed at all those involved in the Early Childhood Development (ECD) sphere and will be a useful resource for ECD practitioners, trainers and planners, as well as parents, carers and home visitors.

In the period before they begin school, which is five turning six years of age in South Africa, children are in a variety of early years settings. Most are cared for during the day in the home, with a minority of these children benefiting from visits from community workers such as home visitors. Some children attend preschools or crèches while their parents work; while still others participate in community-based ECD services such as playgroups or parent groups.

Recent data[1] suggests that only around one in five birth to four-year-olds from lower income groups access formal ECD programmes in South Africa. Access to structured learning opportunities is particularly important for three- and four-year-olds in order to build school readiness, but only about half of this age group attend an ECD centre.

Centre-based ECD provision varies a great deal in quality. Some centres are registered but many are not; and even those that are do not always have well-trained staff or use

programmes that give adequate attention to early learning development. Although early mathematics is covered in the National Early Learning and Development Standards (NELDS), in practice it is often taught, if at all, through a series of disconnected activities that do not constitute a systematic introduction to basic mathematical ideas. The problem is exacerbated by a shortage of appropriate learning programmes and teaching resources that would enable ECD practitioners to translate NELDS into successful classroom strategies. In addition, the training curriculum for ECD qualifications does not give adequate attention to or specific content on early mathematics development.

The challenge

It is well known that South African children are underperforming significantly in language and mathematics.[2] A high percentage of learners are failing to achieve even the minimum expected standards in these core subjects. International data indicate that even when compared with low-income countries in Africa, South Africa compares poorly on tests of mathematics.[3] The reasons for this are complex and rooted in factors that go well beyond the classroom and that affect children's development and wellbeing from birth.

A strong focus on critical issues around pre-school access and attendance, as well as infrastructure and school management in South Africa, has meant that the fundamental questions about ***what*** and ***how*** children are learning have sometimes been neglected. As a result, there has been frustratingly slow progress in improving educational outcomes and in narrowing the achievement gap between children from different backgrounds.

Inequalities in educational achievement are 'locked in' from the earliest years of schooling as many children enter Grade R already behind, having missed out on crucial learning and development opportunities in the early years. The teaching and support that these children receive in formal school is by definition compensatory from the outset, placing an even greater strain on overburdened teachers and scarce resources. The result is that children from more affluent backgrounds who attend well-resourced schools see their relative advantage extend through their school careers, while children from low-income backgrounds who attend poorly resourced schools fall further and further behind.

Young people who leave school unable to apply basic mathematics knowledge to practical tasks are at a severe disadvantage in the job marketplace. Many employers require basic numeracy skills in the same way that they require basic literacy skills, so young people lacking competence in core mathematical areas are likely to find it harder to access a range of career paths. Moreover, everyone requires some degree of mathematical competence to deal

successfully with the everyday demands of life: Whether it is paying a bill, working out the household budget or comparing loan or insurance plans, low levels of mathematics confidence and skills can increase economic vulnerability.

It is clear therefore that if we are to give all children a better chance of fulfilling their academic potential in mathematics, and therefore their full potential as adults, the focus must broaden to include mathematics development in the years before school. Crucially, we must ensure that ECD practitioners have the knowledge and skills to support young children's learning of mathematics.

The evidence

A number of studies have shown that children's mathematics, reading and attention skills when they start school are all linked to higher levels of academic performance in later grades.[4] Children from low-income households who can recite and count to 20 in preschool have the highest mathematics scores in Grade One.[5] Even more significantly, some studies[6] suggest that early mathematics skills (such as counting, number knowledge, estimation and measurement) are the strongest predictor of later overall academic achievement, more so than early reading and attention skills. This supports the wisdom of quality mathematics teaching and learning in the years between birth and five.

Certain types of early years experiences also seem to have a positive influence on the development of children's language and mathematics skills and on later educational outcomes. The large-scale Effective Provision of Pre-School Education (EPPE) project in the UK found that attending preschool supports the intellectual development of all children, but that in order for academic benefits to last for children from low-income backgrounds, it is necessary that the preschool is high quality. The study found that attending a more effective preschool, where early number concepts were introduced, had a significant positive impact on mathematics attainment at ages six and ten. An even stronger link was found between a child's home learning environment and their attainment in reading and mathematics at ages six and ten years – and the ability to deliver an effective home learning environment was not found to depend on parents' qualifications.[7]

We also know more about the different aspects of successful early mathematics development, ranging from the way young children learn and acquire mathematics skills and knowledge, to the methods that teachers employ to teach mathematics in the classroom; and from the role of parents to the effectiveness of various early mathematics programmes and curricula. This research has led to a better understanding and appreciation of early mathematics

concept development and has enabled a wealth of materials and guidance to be produced on evidence-based early mathematics teaching and learning.

What do we mean by mathematics?

Most people associate mathematics primarily with arithmetic – numbers and calculations. The terms 'numeracy' and 'mathematics' are often used interchangeably but the term mathematics should be used to describe the broad subject area of mathematics concepts, skills and applications. Numeracy refers to the everyday uses of mathematics, and includes the ability to reason and to apply simple numerical concepts, such as addition and subtraction. A person with adequate numeracy skills can manage and respond to the everyday mathematical demands of life; in other words, numeracy is an essential life skill in the same way that reading and writing are.

Babies start to make sense of the world in mathematical ways from birth, recognising the difference between small numbers of objects and identifying familiar shapes and patterns in the environment around them. Toddlers and young children continue to develop early mathematics concepts in the early years, through for instance, ordering, matching and classifying objects, and developing and using ideas about shape, space, time and measurement. Although these concepts might not seem mathematical they are important emerging mathematical insights for the growing child. In addition, mathematics provides a powerful means for organising insights and ideas about the world in systematic ways that include describing and representing quantities, collecting information and problem solving.

Mathematics becomes most real and comprehensible for young learners when it develops out of everyday situations and experiences. Mathematical learning can be found and conveyed through many home and classroom activities, and facilitated by teachers, parents and carers in simple ways. In all early years settings, including the home, children can be helped to learn about mathematics through play (for instance, with blocks, sand and water), shared storybook reading, pretend or make-believe games, as well as through their participation in everyday routines and situations, such as going shopping or sharing sweets. Children can also make connections between mathematics and musical experiences, like rhythm and keeping time, and art, when they explore visual patterns or symmetry.

Structured mathematics learning for young children should therefore extend far beyond counting and number concepts, and also include introducing children to the concept areas (such as number, patterns, space, shape, measurement and data handling) and specialised language, which they will require for successful mathematics learning from Grade R onwards.

Language and mathematics

Language is both the cornerstone and mediator of early mathematics learning. At the simplest level, children require specific specialised terminology to understand and use key mathematics concepts. Essential vocabulary includes the words to describe quantities, calculations ('add,' 'take away'), shape, size, measurement ('long,' 'full,' 'heavy'), and position in space ('under,' 'before,' 'between'). Young children also require comparative language ('more,' 'shorter,' 'same') to enable them to sort, compare and reason. Learning these concepts and vocabulary depends on a degree of informal teaching by preschool teachers and parents.

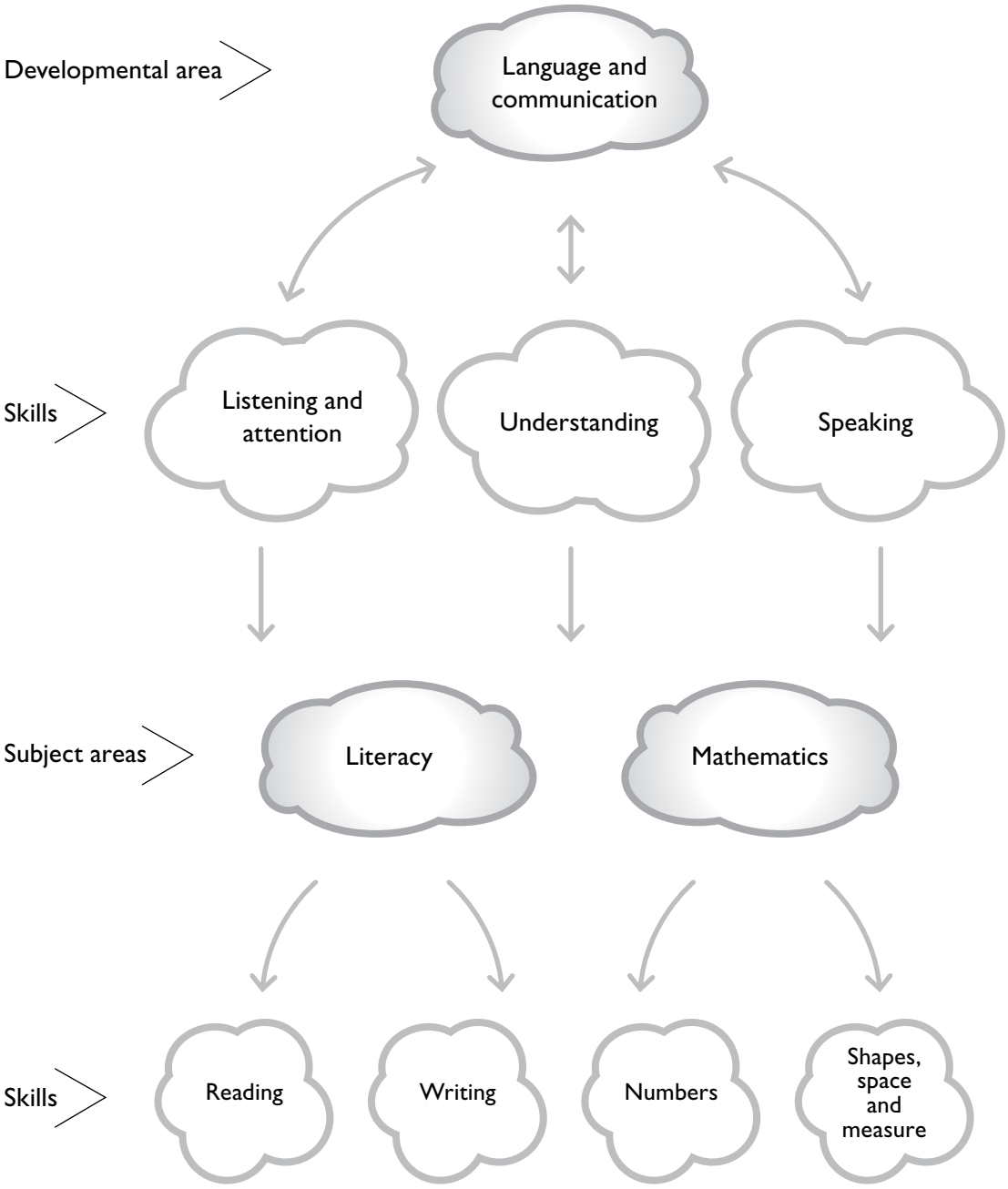
However, the importance of language for mathematics extends far beyond this. A rich foundation of general vocabulary and grammar are required for comprehension, thinking and reasoning skills, as well as for describing, justifying and representing – in other words, for the application of mathematics. For instance, children need the language of prediction ('possible,' 'may happen'), the language of assessment ('how,' 'when,' 'why'), the language of explanation ('I thought the most important clue was…') and of verification ('check,' 'answer,' 'correct'). Children also need to be able to follow and comment on an adult's or another child's line of reasoning and to formulate questions. To encourage this kind of general language development, children need plenty of opportunities to interact and communicate with adults, and to talk about their ideas and reasoning.

Mathematics is a language of symbols, so children also need to start to learn and use the symbols (for example 1, 5, +, =) that convey mathematical ideas. Through drawing and early writing children begin to understand that written signs and symbols represent words and things.

In all these ways, both language and mathematics are about conveying the meaning of ideas and therefore depend on and promote each other – as children explore mathematics and extend their thinking, so their language will grow. Figure 1 draws on the conceptual framework for the UK's 'Statutory Framework for the Early Years Foundation Stage' and accompanying guidance, to illustrate that without competence in language (listening, understanding, speaking) children will be unable to fulfil their potential in the spheres of both literacy and mathematics.[8]

During the early years, research suggests that children should be encouraged to use their home language as much as possible and that this fosters both overall linguistic ability and cognitive development. However, we know that in South Africa, many children are in early years settings where they are learning in a second language. In these circumstances, mathematics learning can be a valuable tool for promoting general language development in particular for second language learners, through the types of interactions it encourages and the opportunities it creates for children to discuss their reasoning and share ideas.[9]

Figure 1:
The essential role of language in early learning



The following chapters examine mathematics development in more detail and provide practical ideas and suggestions for the teaching and learning of early mathematics. In Chapter Two we consider the range of ways in which young children learn. We then look at what constitutes 'mathematics' in the early years in Chapter Three, and explore the different learning strands. Chapter Four sets out the learning highlights and possible activities for the different age groups. Practical examples are used throughout, to bring to life the concepts and methods being explained.

Chapter 2

How young children develop and learn

CHILDREN LEARN IN MANY different ways and at different rates. However, there are certain characteristics of effective learning that apply to all children and that all ECD practitioners and parents need to be familiar with. These ways of learning can be promoted as much through everyday contexts and routines as they can through specialised learning activities. They apply to all subject areas, and below we give specific examples that are relevant to mathematics.

Trusting relationships

Early learning takes place most effectively in the context of a trusting adult-child relationship where children feel safe and secure. Developing supportive and dependable relationships will help children learn effectively every bit as much as having the right resources. As such, it is important that children feel that their contributions and questions will be valued and engaged with respectfully. This helps to give children the confidence to explore new ideas and to express themselves with the support and encouragement of parents and carers as they share their learning experiences.

IN PRACTICE…

Create a safe and enabling environment where children can engage in risk-free exploration.

Pay attention to whatever the child is paying attention to and be prepared to follow the child's lead.

Show that you value what they have chosen to do by praising their efforts, answering their questions and not being critical.

Being curious and exploring

Babies and young children are naturally curious, and use all their senses to explore and learn about the world. Children have a natural ability to concentrate when they are interested in something and the richest learning moments are often those initiated by a child. Babies may use gestures and simple sounds to indicate interest and engage an adult in their exploration of their immediate environment. Adults can foster learning by following a child's interest and then extending and elaborating on the focus of interest in order to sustain the engagement and thinking. Careful use of questions and prompts will also help extend children's natural curiosity.

IN PRACTICE…

Provide opportunities for children to explore mathematics concepts by connecting mathematical ideas to their play and routines and the activities they most enjoy.

Offer new mathematical language through comments like, ***"If you take another two balls out of the bag, you will have four altogether."***

Use questions and suggestions to help them reflect on what they are discovering and to consider the next step: "***I wonder whether the car is too big to fit inside the box?"***

Encourage children's 'what?', 'if?', 'why?', 'how?' questions, and help them to work out answers for themselves.

Learning through doing

Young children learn by playing and doing, and by being exposed to situations in which new concepts and ideas are made real and meaningful by linking them to everyday experiences. As with adults, children absorb more information when they are involved in hands-on activities, particularly if they are fun and linked to the child's natural interests. This type of learning is enabled by helping children to make connections between what they already know and something new that they are exposed to.

Children love to be involved in 'grown-up' activities like tidying up or preparing a meal, and these can provide fruitful learning opportunities for adults to introduce mathematical ideas. This kind of guided participation gives children the chance to be apprentices as they learn new skills, and to see how their emerging knowledge can be applied to everyday tasks and situations.

Imitation is another way of learning by doing. It is not the same as copying, but a way in which children observe, process and then reconstruct an event. Many learning situations are created when young children seek to imitate older children or adults.

IN PRACTICE…

Include children in activities that involve numbers during daily tasks. For instance, let them help with counting items when shopping, weighing ingredients for cooking, and sorting and matching the washing.

Allow children to notice and correct mistakes themselves, and talk about what they are thinking and doing.

One example is the regular task of laying the table for a meal:
Mom asks, "Please help me finish setting the table for dinner. Have I put out enough plates? Let's count them. Right there are six plates. Do we need any more?" Child says the name of each member of the family and counts on his fingers. He says, "Yes, Granny doesn't have a plate!" Mom says, "We need to get a plate for Granny. Can you set out enough cups for everyone? How many cups did you put out? Do you have the same number of cups as plates? Let's count them."

Talking, listening and questioning

While language underpins all learning, specific communication skills and types of interaction lie at the heart of the learning process. Children need to know how to be part of a conversation, to listen to what has been said and to respond to and build on it. These skills will help them to work with others to check the meanings of concepts, solve problems, and plan and evaluate activities. The more children are encouraged to be part of a conversation and to articulate their questions and ideas, the quicker their language and thinking skills will grow and facilitate new learning. Extending children's comments through giving them the words to express their thinking and ideas, indicating interest, asking questions and giving constructive feedback can help to achieve this kind of confidence in thinking and communicating.

IN PRACTICE…

Provide abundant opportunities for children to talk about what they have done and to explain why they did it.

Listen respectfully, for example by showing interest in their explanations and ideas, and being at the child's level and making eye-contact.

Use a range of questions, including open-ended questions, to extend and consolidate children's thinking and to promote the use of number language, comparison and estimating. For example, ***"You have made lots of snakes. Which do you think is the longest? How can we check?"***

Look for opportunities, such as games and races, to model the language of numbers, quantity and measurement through simple extension.

Playing

Children's play provides rich opportunities for learning. Play can mean many things: physical activities outdoors; imitating adult activities and routines in games; playing with sand or water; pretend play and make-believe with friends or alone; and structured games such as snakes and ladders or dominoes. All these activities can be facilitated by creating a safe environment in which children are encouraged to take the lead and to cooperate with each other, and where adults are prepared to participate in children's choices and imaginings. Simple resources can help to stimulate different kinds of play, many of which can be homemade.

Pretend play is particularly important as it involves the use of objects to stand for or represent other objects, and is therefore a vital first step towards understanding that things can stand for other things (symbolic representation) and towards abstract thought. From play, children start representing things in drawings and begin to understand the link between written symbols and actual things – such as, the symbol '5' stands for five objects.

IN PRACTICE…

Join in and share activities with children as they play, talk to them and think aloud.

When you participate in a child's game, talk about what you are doing, ***"I filled three cups with water, one, two, three, and now I've filled one more, so there are four."***

Notice how children talk about their ideas about counting, combining and sharing during their self-initiated play, and repeat their findings back to them, for example: ***"You counted out five red beads and then counted out five blue beads. Let's count how many beads you have. That's right ten beads."***

Help children to develop symbolic thinking through pretend play, by suggesting how one thing might represent another – for example, chairs in a row becoming a train.

Stories and books

Storytelling and books present many and varied learning opportunities, including the acquisition of new language. Shared storybook reading and telling stories provide occasions for children to discuss ideas, concepts and predictions. Wider learning can be introduced into storybooks, for example, by encouraging children to count objects on the page or to use spatial language ("***The goats went over the bridge.***"). Through storytelling, children also learn about sequencing information in order to describe events in a logical way.

IN PRACTICE…

Whether at home or in a formal ECD setting, seek to create an environment rich in storytelling.

Look for opportunities in shared storybook reading to explore and explain new mathematics concepts.

If you do not have access to storybooks, make time every day to tell real and make-believe stories and encourage children to do the same.

Songs, rhymes and movement

Songs and rhymes are a fun way of introducing and reinforcing mathematical concepts, for instance, through counting songs and number rhymes. For many children, particularly those learning in their second language, singing is an enjoyable and non-threatening way of learning language. Songs and rhymes also give children opportunities to hear patterns and repeated sequences in the music and rhythm.

Children use their own bodies to orientate themselves in space and learn about their world. Movement and dance are therefore useful ways for children to learn about position and direction, as well as repeated patterns.

IN PRACTICE…

Use number rhymes and songs that focus on counting and calculating.

Give children opportunities to 'act out' the rhymes and physically move – as toys in a bed or frogs on a log. ***"If there are ten children in the bed and one falls out, how many are left?"***

Ask children to clap a musical pattern or a rhythm. Drums, marimbas and other instruments can also be used.

Invite children to represent patterns in movement: jump-jump-clap, jump-jump-clap, jump-jump-clap-clap.

Problem-solving

Young children's natural curiosity encourages them to seek out solutions for problems that they encounter in their day-to-day experiences. When problems arise from contexts that are personally meaningful and relevant to a child, their interest is sparked and they are likely to be highly motivated, persisting in their enquiries through reasoning and experimentation. As adults assist children to make predictions, try different strategies and observe and evaluate outcomes, children gather information and develop new understanding.

Much of young children's problem-solving is based on trial and error. It is therefore important that they are not scared of making mistakes, that early learning attempts are not continually corrected, and that effort and persistence are celebrated. Some children become easily frustrated and do not develop the ability to persevere and sustain problem-solving. Adults can support this process by 'scaffolding' or helping children to think about cause and effect, eliminate unsuccessful strategies and use the knowledge they have already gleaned to lead them to a solution. Children become better and more confident problem-solvers through learning to reflect on similar situations where problems were solved, to generalise strategies used and adapt these to the new situation.

IN PRACTICE…

Observe when a child is grappling with a problem and give them the space to express themselves and to ask for help if they need it. If necessary, prompt the child to explain what the problem is and what they think they should do to solve it.

Affirm children's efforts and the process rather than the outcome, and help them to identify and value the learning in 'unsuccessful' outcomes as well as in successful ones.

Encourage children to experience an activity as one of shared exploration where a problem is solved together, rather than one where the adult holds the right answer, which the child must discover.

Be specific in your praise, especially noting persistence, concentration, trying new approaches and responding to learning.

Avoid correcting every mistake, such as when a child misses out a number when counting, or gets them in the wrong order – it is more important at this early stage that they enjoy the activity.

Repeating

Children need and enjoy repetition of activities. They are happy to hear the same story or to play the same game over and over. Repetition helps to reinforce learning before an activity is extended to encompass new ideas. It also helps children to generalise by repeatedly testing an idea in a way that builds evidence for a solution. When children play the same game a few times, they develop the confidence to try new strategies and take their understanding to new levels. It is important therefore to providing sufficient time for exploring and for maintaining key ideas, and devoting enough time to each activity to allow for repetition.

IN PRACTICE…

Re-tell stories to provide opportunities for children to practice using their new mathematical vocabulary. This is particularly important for children learning in a second language.

As children become more familiar with a story, allow them to participate in the re-telling and to experiment with the new language.

Be patient as children seek to repeat activities and games and resist trying to hurry them on to something new.

Chapter 3

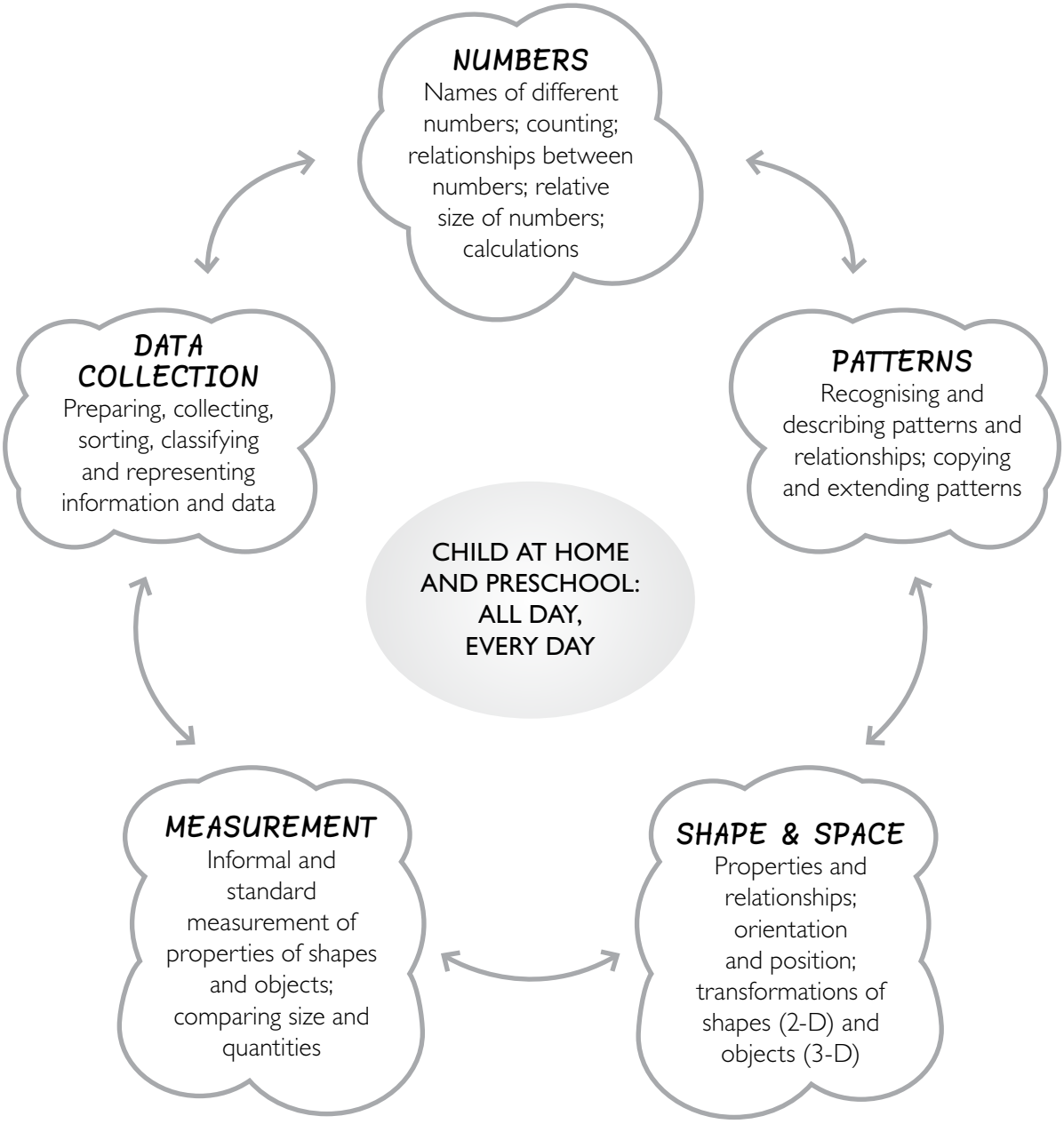
Mathematics concept strands

CHILDREN'S UNDERSTANDING OF mathematical concepts and ideas increases significantly in the years before school. Everyday situations and the ideas they share with adults about these experiences form the basis of much of this learning. For example, children begin to represent numbers and mathematical ideas during daily routines, using objects, in play, and by drawing and writing symbols. As early as age three, children can hold up fingers to indicate a quantity, and repeat a series of numbers, such as ***"one, two, three, six, ten."*** Although they are not always accurate, this is the basis of mathematical behaviour and exploration. Parents and carers often stress numbers and use counting as part of their everyday interaction with babies, even from their first days.

During this period, children are building important foundations for school mathematics. For instance, they will start to recognise and name shapes that have different sizes and orientations, and to use directional words such as 'up,' 'over,' 'forward.' As they compare toys and objects, children will develop language and concepts of size and measurement. Young children's ability to solve different kinds of problems, to sort and classify and to make predictions will also become increasingly developed.

In this section we offer a broad overview of the mathematical knowledge strands that are appropriate for children aged between birth and five years, and present some of the foundational ideas that build on them. These ideas are important preparation for school mathematics and link to the five content areas presented in the Foundation Phase Curriculum and Assessment Policy Statement (CAPS),[10] which are shown in the diagram below.

Figure 2:
Early mathematics concept strands



Although these strands of mathematics reflect particular areas of development as presented in the South African school curriculum, they are closely linked and often overlap during activities. Each of the five strands is considered in more detail below.

Numbers

As children encounter numbers in their everyday surroundings, they begin to discover that numbers can be used differently in different situations. In this way children are learning different meanings and uses for number. For example, the number five can be:

- a magnitude, or 'how much': *"I have five sweets."*
- an order number: *"She is the fifth person in the row."*
- a measure number: *"He is five years old."*
- a label number: *"We live at number five."*
- a calculation number: 2 + 3 = 5

Numbers give us specific, detailed information about collections and quantities, and communicate how many things there are or how much of something there is. Numbers are abstract concepts – they are not objects themselves but describe something about other objects. Just as the word 'green' describes the colour of an apple, so the number 'six' might describe the number of apples in a collection.

Counting

Counting is a complex skill that needs a lot of practice, and develops in contexts where children count real objects. Often this development starts with imitating the counting of older children and adults. Initially young children do not fully understand the meaning of the words and might skip numbers in a counting sequence.

Reciting a rhyme or series of numbers orally means repeating the numbers from memory. This is different to counting to find out 'how much.' To count 'how many' children need to realise that each object in a collection gets a number name ***("one, two, three, four…"***) and that you count each object only once.

With lots of practice children begin to understand that counting involves the following aspects:

- Matching one, and only one, counting word to each object in the collection being counted (one-to-one correspondence).
- Repeating the counting sequence and number words in the correct order.
- That the last number word they say when counting a collection of objects stands for the total of the whole collection (cardinal principle).
- That we can use a number to indicate a position of something in an order or sequence: 'first,' 'second,' 'third…' (ordinal principle).

Recognising a small collection of objects without counting

People have the remarkable ability to recognise the number of items in small collections of between two and five objects immediately without counting them. This ability is called 'subitising' and can be encouraged by playing with dice and dominoes. Children love to play games that involve quickly showing a small number of objects and hiding them, then making them pop up again and asking how many there were.

Estimating

Counting is not limited to finding the exact number of a collection. Children also need to develop estimation skills so that they can say 'about how many' objects there are in a collection, and use terms such as 'a lot,' 'few,' 'more' or 'too many.' This lays the foundation for estimation skills, and also provides an opportunity for children to use counting skills to find the actual number in or specific differences between collections.

IN PRACTICE…

Mrs Erasmus brings a jar of sweets to her playgroup. During morning circle time, she shows the children the jar of sweets. She asks the children to guess how many sweets are in the jar? The children guess different amounts: Nozuko says ten, Marie says eight and Mohammad says 15. Then, together they count the sweets one by one. There are 20 sweets. They discuss whether there are enough for everyone to have a sweet. Some children say yes and others say no. After lunch each child takes a sweet from the jar. They then discuss the fact that there are 20 children and 20 sweets so everyone has one – except Mrs Erasmus! They agree that they will need one more sweet or 21 sweets for everyone to have one.

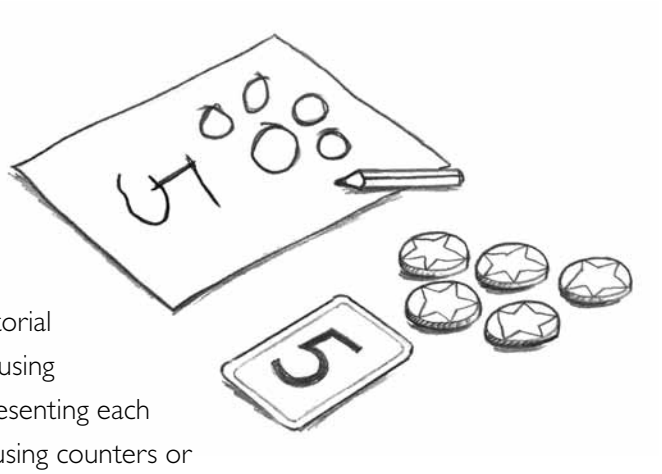
Calculating

Through calculating children learn how to operate with numbers. When children are involved in counting activities they will begin to compare collections of objects. They might line up the objects or set them out in such a way that they can compare and count them in order to determine which group has more or less. Through these experiences they will develop an understanding of equivalence and non-equivalence.

Through activities and experiences that involve breaking up, adding to and comparing collections children begin to develop the concepts of combining (adding) and taking away (subtraction). Children are also exposed to addition and subtraction during their everyday games and activities, for example when they play 'shops' together or have to share toys. For subtraction children need to take part in practical activities that involve 'taking away,' in other words, finding how many are left in a collection of objects when some have been removed. Initially children will use counting-based strategies to solve problems involving addition or subtraction – for example, counting all the objects in two collections to reach a total amount when the two collections are combined, or counting how many coins are left when some have been given away.

Multiplication, division and fractions are rarely explicit in early years mathematics, but children often participate in activities that involve these concepts – for example, when sharing seven apples between three people, they will discover that each person gets two apples and there is one left over. Repeated addition (combining) and repeated subtraction (taking away) are ways in which to help children understand the concept of multiplication and division. This also helps to establish relationships, through experience, between addition and multiplication and subtraction and division, which need to be understood later on at school.

Representing numbers

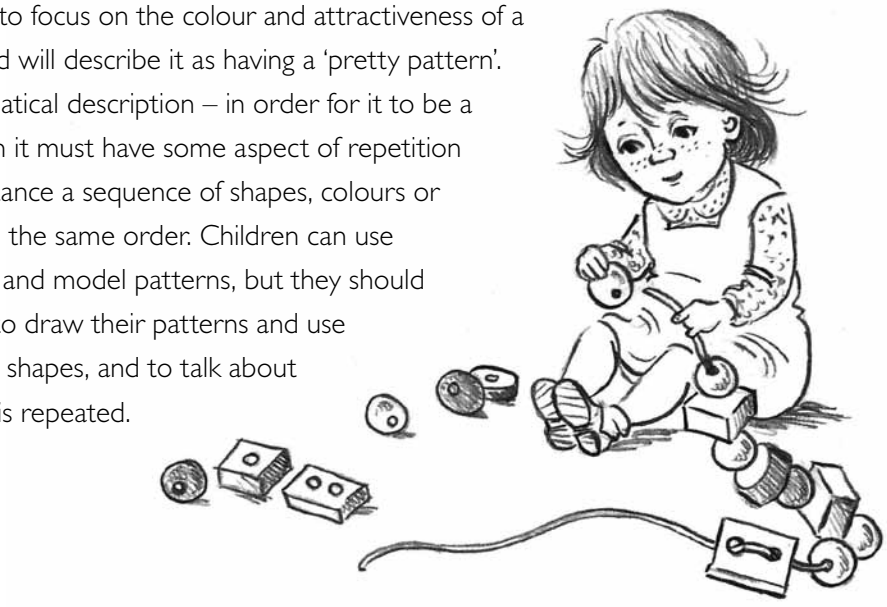


Children begin to represent numbers by showing their fingers and then gradually start to use other methods, such as pictures or symbols. Their first recordings of numbers and calculations might involve mark-making or simple pictorial representations of what they have done using actual objects. Children move from representing each of the physical objects in a collection to using counters or markers that stand in place of the real objects. For example, for an addition problem, children might record the outcome when two groups of objects are combined into one group by using counters or drawing the solution. This leads to the next level of understanding in which children use symbols to represent numbers. To assist this process, children should be encouraged to talk and write about their work in their own ways.

Pattern

Mathematics involves the study of patterns and relationships. Understanding and working with patterns lays essential groundwork for algebra and is therefore a vital element of early years mathematics. As children try to make sense of and understand the rules that govern their world they encounter patterns everywhere: in people's behaviour; in cycles, time and routines; in nature; in music and art; and in their built environment. And of course children will notice numerical patterns in sequences of numbers.

Young children tend to focus on the colour and attractiveness of a picture or object, and will describe it as having a 'pretty pattern'. This is not a mathematical description – in order for it to be a mathematical pattern it must have some aspect of repetition or symmetry, for instance a sequence of shapes, colours or numbers repeated in the same order. Children can use objects to represent and model patterns, but they should also be encouraged to draw their patterns and use different colours and shapes, and to talk about the way the pattern is repeated.



IN PRACTICE…

To help children recognise the pattern relationship between numbers, create a painted number track outside or use number lines at home or in the classroom.

Ask questions that draw children's attention to the sequence of numbers, and how they are placed in relation to each other. For example, ask children to jump from one to two; ask them how many steps or jumps they took and which number comes next; ask them how many steps they will need to get to four.

Provide opportunities for children to observe, describe and discuss patterns, using the following activities:

- string beads of alternating colours
- draw or paint striped designs
- use blocks of different shapes and sizes to create repeating sequences
- clap rhythms when saying words
- ask children to repeat a pattern of physical movements

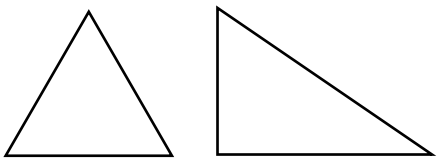
Symmetry is a form of pattern and also links closely with the strand of shape and space. Children can be helped to notice symmetrical patterns all around them, in nature, in buildings, in paintings and objects. In the early years, symmetry is easiest understood as 'reflection' or 'mirror.' Children can explore this concept by cutting shapes and pictures in half, or by drawing a picture on one half of a piece of paper using wax crayons, then folding the paper and rubbing the area behind their drawing and seeing the exact copy of what they have drawn reproduced on the other half of the page.

Shape and space

Young children orientate themselves in space on the basis of their own bodies. They begin to explore the relationship between themselves and objects, and later between objects and other people. Babies reach and grasp objects in close proximity, and then gradually start to move around exploring their environment using all their senses. They explore the consequences of pushing, pulling, rolling or turning different objects as they play with them, and in so doing develop a sense of themselves in relation to the objects around them. At the same time, they learn the limitations of their own physical movement as they climb over and under chairs, into boxes, hide behind trees or look down from steps. As they participate in physical games and activities, such as treasure hunts and mazes or simply climbing into bed, with the help of adults, children develop the vocabulary to describe space, position and direction, such as, 'above,' 'in front of,' 'between,' and 'backwards.'

During play, children will also observe the shapes of everyday objects and toys and begin to classify them by, for instance, putting all the leaves in a box or separating the balls from the blocks. This kind of classification shows that children are learning to discern physical differences between objects. Children will recognise symmetrical and regular shapes and patterns more easily than irregular ones. A shape is regular if all the sides are the same length. These two shapes are both triangles, but only one is regular:

Children will often only recognise the first shape as a triangle because the three sides are equal length, in other words regular. The other triangle is irregular because the sides are not all the same length.



Children live in a world of objects that have a physical form that is three-dimensional (3-D) and often irregular. It is therefore important to talk about these types of shapes as well as regular two-dimensional (2-D) shapes such as circles and squares. Initially children are able to distinguish between shapes based on their ability to use visual matching. Gradually young children's understanding of shapes expands and they begin to recognise characteristics or simple properties of shapes, such as how many sides they have. Children can be helped to explore the properties of a wide range of regular and irregular shapes, through talking about differences and sorting shapes according to their properties – for example, the number of sides or corners a shape has, or whether the shape can roll or slide.

Measurement

During the course of their daily activities and conversations children encounter measurement, as they compare size and explore length, weight, volume and the passing of time. Conceptual understanding of different measures develops gradually and grows out of children's day-to-day experiences. With friends they might take the biggest piece of bread or compare height or find out who has the smallest foot or who has made the tallest tower. They may measure out ingredients for cooking, pour water or sand from a jug to see how many cups can be filled, or compare how heavy a bag of sugar and a box of oranges are.

Measurement is an important aspect of mathematics and has strong links with the other strands. Measurement is about making comparisons, about determining the size or amount of something by comparing it with an object of known size (for example, hands or a pencil), or a standard unit of measure (such as a centimetre or litre). Measurement in the early years is informal and usually involves estimating and comparing non-standard forms of measurement. Children should be encouraged to do their own measuring so that they can deepen their understanding of the different types of measurements.

In the real world we use our skill of estimation far more than we use measuring tools. When babies move around and explore their environment they estimate the distance and space between objects, like furniture. As children grow they will use estimation skills to estimate the amount of water to pour into a glass so as not to overflow or spill the water. Good estimation

skills require a lot of prior experience and practice in different situations. Eventually, children will need to learn how to use measuring tools and learn to measure as accurately as possible. Importantly, measurement requires the use of an object or unit to measure and involves using numbers and applying number operations in real situations. The focus in the early years, however, is on developing the concepts of measurement and using appropriate language to compare and describe measuring in children's everyday lives.

Measurement is linked to a key concept in early childhood development called 'conservation.'[11] Children who cannot conserve will think that a straight line is longer than a wavy line of the same length or that a taller container holds more water, while others will say that the fatter one holds more water, even if they have seen the same amount of water poured into both containers.

Time is an aspect of measurement that is particularly difficult for young children to understand. This is partly because adults do not always use the language of time accurately. For example, a child may hear an adult say ***"I will be there in a minute,"*** but then take much longer than that. Young children respond in the moment and therefore recalling past events in order or predicting future experiences is more difficult for them. Children need to become familiar with the language of time so that they can talk about the order in which a sequence of events occurs, for example, 'before,' 'yesterday,' 'later.' Time is also another way of measuring movement, so children need to become familiar with concepts like 'fast,' 'quick,' and 'slow.'

IN PRACTICE…

Length

- Provide children with opportunities to sort objects according to length and height, to make comparisons and describe what they have found.
- Key vocabulary includes 'long,' 'short,' 'tall,' 'thin,' 'thick,' 'narrow' and 'wide.'
- It is helpful for children to be able to draw and paint their understanding of these concepts and then use these as the basis for discussion. This will encourage children to make more accurate observations, even if they are not yet able to represent these.

Weight

- Let children use their hands to estimate and compare whether objects are heavy or light.
- Create occasions for children to use balance scales and experiment with weight of different objects and substances, through formal activities and through games like playing 'shops.'
- Introduce activities that help children to discover that the size of an object is not directly related to its weight, and that not all heavy objects sink or light objects float.

Volume

- Children use the concepts of empty and full when they are filling or emptying containers, packing away toys into boxes and even when they are eating their food.
- Help children to fill containers with different substances and compare their capacity by using the language of volume. For example, ***"How many cups of water do we need to fill the jug? Why do we need fewer milk cartons of water to fill the jug?"***

Time

- Tell stories that involve sequencing events and talk about routines at home and preschool and what happens at different times of the day and night.
- Help children to learn the names of some units of time, such as minutes and hours, and the names of the days of the week.
- Introduce children to a clock as a way of measuring and telling the time.
- Relate notions of time to other measurements, for instance, ***"The heavy stone falls quicker than the light feather."***

Data collection

Data collection involves collecting, sorting, recording, interpreting and presenting information. Children constantly go about their daily activities sorting and classifying objects around them in different ways. They put toys into groups of different colours and sizes, they pack and unpack household items, sorting them into piles of different shapes and uses. Gradually children are able to classify according to different features or criteria and are able to give reasons for why they grouped objects in a certain way and consider other ways of organising their information. As children explore and talk about how they are gathering, organising and sorting 'things' around them, they develop these classification skills.

In the early years, children can be assisted to make the collection of data or information purposeful to help them achieve a goal, answer a question or solve a problem. There are some helpful considerations that can guide young children's data collection:[12]

- recognising the shared attributes of objects, like colour, size and shape
- comparing more than one object
- matching two or more objects
- representing and interpreting discovered data using pictures

IN PRACTICE…

Ask a group of children whether they prefer cats or dogs. Draw a dog and a cat on a large sheet of paper and let each child make a mark in evenly spaced boxes next to the one that they prefer. Pose questions to the children about the 'pictograph' that they have created: ***"Do more of you like cats or dogs? Which is the favourite animal? How do we know?"***

This activity can be made more complex by exploring questions with a greater range of answers, such as children's favourite foods. This might involve introducing an extra stage for children to categorise the information (for instance, fruits, vegetables, meats, sweet things).

Chapter Four

Learning highlights and examples

The table below provides practical examples of the development of the mathematics strands in the previous chapter. It indicates what the 'learning highlights' might be of children in different age groups and suggests ways in which adults can support development and learning.

It is important to note that although children progress through a generally predictable developmental pathway, each child is unique and they may not proceed through these stages in the same way or at the same time. Each child's development is greatly influenced by factors in their environment and their personal experiences. The information about 'learning highlights' should therefore not be used as a checklist or treated as necessary steps. The age boundaries overlap because they indicate a typical rather than a fixed range of development. In practice, parents and teachers should be responsive to the pace and preferences for learning of each individual child.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Numbers	
Birth-22 months (birth to almost 2 years)	• Starts to use number words even though they do not understand what they stand for. • Begins to understand concepts such as 'more' and 'enough.'	• Sing songs that rhyme, repeat, or have numbers in them, e.g. ***"One, Two, Buckle My Shoe"*** or ***"Two little dickie birds."*** • Use numbers incidentally during daily routines, such as getting dressed ("***two shoes and two socks"***) or during play time ***("teddy has two eyes and one nose").***
18-36 months (1½ years to 3 years)	• Starts to understand the words 'one' and 'two' and may hold up two fingers to show you. • Tries to recite number words in sequence but may get numbers out of order. • Is aware that two is more than one and one is less than two – the beginning of understanding addition and subtraction. • Follows simple directions involving numbers, e.g. "***take two biscuits"*** or ***"pick one."*** • Uses size terms, like 'same' and 'more,' to make comparisons in games and everyday activities.	• Sing number songs and rhymes together, from a range of cultures and languages – include actions, clapping and marching. • Encourage children's counting efforts without correcting them – through repetition, they will learn the correct order. • Create opportunities for counting – ***"One apple for Sam, one for Zinzi and one for Dad, we need three"*** – or letting children count and hand out the cups during snack time. • Use estimating terms such as 'about,' 'nearly,' 'many,' 'same,' 'a few' during daily activities. • Play games which relate to number order such as hopscotch. • Count together when reading books, pointing to people, animals or objects on the page. • Introduce words such as 'first' and 'last.'

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Numbers	
30-48 months **(2½ years to 4 years)**	• Counts up to ten or more. • Knows that each number refers to an item in a set and starts to use some number names accurately and to count in meaningful ways in play and daily activities. • Counts to compare small quantities. • Can separate a set into two equal groups. • Begins to recognise some written numerals.	• Try a backwards counting song, like ***"Five in the bed and the little one said roll over."*** • Count real things, such as counting down on the calendar the number of days or weeks until a child's birthday. • Provide many opportunities to develop one-to-one correspondence, e.g. using egg boxes and play dough eggs, put one egg in each space and use questions such as, ***"How many more eggs do you need to fill your egg box?"*** • Give children real reasons for counting such as handing out paper and crayons for each child in their group. • Play games which relate to subtraction and addition, such as skittles. • Point out numbers around you, such as house numbers, car number plates, prices in shops or numbers on coins.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Numbers	
42-60+ months (3½ years to 5+ years)	• When counting items in a collection, labels each object with just one number word to determine the total (one-to-one correspondence), up to 20 items. • Knows that the last number they say 'in the count' represents the total number of objects, without needing to start counting the objects all over again. • Knows what number comes after a given number in a sequence up to ten. • Given two numbers between one and ten, can tell which of the two is larger. • Can recognise small quantities of up to three or four objects at a glance without counting, for example, recognising the number patterns on dice. • Can add and subtract using numbers up to four. • Can share objects equally between more than two groups. • Uses words 'first,' 'second,' etc. to indicate the position of something in a sequence. • Recognises written numerals 0 to 9. • Starts to use formal comparative terms such as 'fewer,' 'greater than' and 'equal to.'	• Use opportunities to compare and add the number of items in collections of objects, e.g. ***"Let's count how many blocks you have in your tower? Let's county how many Molly has. Now let's count all the blocks (both yours and Molly's) to see how many we have."*** • Display and read interesting and varied books about numbers. • Play simple board games, dice games and dominoes, using large dice with bold number patterns to start with. • Encourage games that include counting money such as a pretend shop. • Encourage children to think about logical answers to simple problems or questions, e.g. ***"I wonder how many sweets will be left if I give two to Gina? What do you think?"*** • Play games that encourage children to look for numbers in their everyday environment, such as on number plates, in shops and on money. • Make number lines available and show children how to use them and incorporate them into their games. • Give children opportunities to practise estimating, e.g. during water play ask, ***"How many cups of water will it take to fill the small jug. Let's check to see? Look at this big jug, will it take more cups of water to fill it? Let's check to see."*** • Play guessing games such as, ***"How many oranges do you think are in the basket?"***

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Pattern	
Birth-22 months (birth to almost 2 years)	• Observes, hears and feels many different patterns (sees shapes and shadows, hears a repetitive lullaby, feels the rhythm of people walking or dancing when they are carried). • Shows a preference for patterns with large elements to start with, then enjoys increasingly complex patterns. • Begins to work out that there is an order to their days, for instance, they go to bed when it is dark.	• Introduce routines to help babies develop a sense of the order of things, and to anticipate the pattern of their day. • Play games involving simple patterns using their bodies, e.g. ***"clap hands, touch knees, clap hands, touch knees, clap, clap, clap."*** • Listen to and sing songs and rhymes with repetition. • Point out and touch bold patterns in the house and outside e.g. stripes on the curtain or footprints in the sand.
18-36 months (1½ years to 3 years)	• Becomes more aware of patterns around them, both man-made and natural. • Becomes familiar with patterns in their routine, e.g. that night and day follow a continuous cycle. • Can describe a simple sequence of events, such as the steps they follow in getting ready in the morning: ***"First we wake up, then we wash, then we get dressed, then we have breakfast."***	• Draw children's attention to daily rhythms such as day and night, morning routine or meal times. • Encourage children to notice different patterns around them either man-made, such as tiles on a roof, or in nature, such as spots on a lady-bug, and how they are similar and different. • Collect pictures that show the use of patterns from a variety of cultures such as Arabic rug designs or shwe-shwe fabric.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Pattern	
30-48 months **(2½ years to 4 years)**	• Creates patterns with objects and art materials. • Identifies, copies and extends simple visual patterns that repeat, including colour and shape. • Begins to understand the pattern (sequence) of numbers. • Can follow and make their own sound patterns either with their body or with different instruments. • Understands the idea of symmetry as 'reflection' or 'mirror.'	• Give children opportunities to draw patterns or create them with a variety of objects. • Show children simple patterns – such as shell-stone, shell-stone; red-yellow-blue, red-yellow-blue; or circle-square, circle-square – and ask them what comes next. • Ask children questions about the sequence of numbers and how they are placed in relation to each other – use number lines/ tracks that children can move along. • Make your own musical instruments to play simple tunes on and help children to follow the patterns of notes and rhythms. • Help children to notice symmetrical patterns around them, and to explore symmetry by cutting shapes and pictures in half.
42-60+ months **(3½ years to 5+ years)**	• Is increasingly aware of more complex patterns in nature, e.g. detailed leaf or flower patterns and animal life cycles. • Uses words like 'striped,' 'checked' to describe patterns. • Understands and uses the concept of symmetry. • Matches their body movements to the rhythm and mood of music. • Describes patterns of events and makes logical connections and generalisations.	• Build on children's natural ability to recognise number patterns. Include counting in twos, ***"Let's count all the legs in the room – two, four, six, eight, ten."*** • Encourage children to collect and notice patterns around them, e.g. tracing complex leaf patterns or recording the cycle of caterpillars transforming into butterflies. • Help children to create symmetrical artwork and models and discuss what makes them symmetrical. • Provide music and props for children to move creatively in plenty of space and suggest new patterns of movement. • Notice when children make predictions and generalisations and ask them to describe their reasoning.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Shape and Space	
Birth-22 months (birth to almost 2 years)	• Understands that when an object is hidden from view, it can still exist and searches for things that are out of sight. • Explores and solves spatial problems by moving over, around or under objects. • Enjoys filling and emptying containers. • Can do simple insert puzzles when the puzzle pieces show whole objects or simple shapes.	• Play games together by hiding behind a chair ('peek-a-boo') or hide a toy under a cushion and let the child find it. • Encourage children's spatial sense by letting them climb in and out of boxes, on or around furniture – going under, over, around, through, into and out of different things to experience themselves in space. • Give children empty plastic food containers or pots and pans to stack on top of or inside each other. • Look at books together showing different objects and shapes, such as 'Big truck and Little truck.'
18-36 months (1½ years to 3 years)	• Identifies different shapes by sorting them and starts to name some shapes. • Stacks a set of rings on a peg by size. • Puts together simple puzzles, where each piece forms part of the whole, starting with two and three piece puzzles. • Stacks objects to make a tower and uses blocks to get a sense of how objects can be ordered in a row – vertically or horizontally. • Becomes familiar with concepts such as 'on top' and 'underneath' or 'in' and 'out' through everyday activities and play.	• Provide opportunities for children to play with shapes, rings and pegs either using toys or everyday objects. • Point out shapes encountered during activities and in the environment (e.g. on traffic signs or during meals) and talk about how the shapes differ. • Cut out shapes together and use them to create a picture. • Provide opportunities for doing puzzles in a supportive and non-pressured environment. • Compare different handprints – big and small and how they compare to a child's footprint. • Encourage children to explore building blocks or constructing with different objects, using colourful blocks, wood off-cuts or objects around the home. • Give children opportunities to open and close doors, pots with lids or boxes around the house, and encourage them to place objects in and out of these containers. • Use music and dance to create fun opportunities for children to develop spatial awareness and positional language.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Shape and Space	
30-48 months (2½ years to 4 years)	• Enjoys representing their world, including their body image, through drawing. • Can describe relative position and direction using words such as 'up,' 'down,' 'in front,' 'next to,' 'outside,' 'between'. • Becomes aware of a variety of 2-D and 3-D shapes in their environment. • Builds simple puzzles. • Becomes aware of how things fit together.	• Provide plenty of experimental, non-critical opportunities for children to draw. • Help children to become more aware of their own body, ***"Let's touch your head, it's at the top, and feet are at the bottom, and we both have two hands."*** • Use the appropriate directional words with children in different situations, including with their own bodies ***("Can you stand in front of the chair?"***) and with objects ***("Can you pass me my shoe it's under the table?"***). • Choose a shape for the day and look for that shape wherever you go together, include both flat shapes and 3-D shapes, such as triangles, rectangles, cubes, cylinders and spheres. • Discuss shapes outdoors: clouds, flowers, buildings and trees. • Provide different age-appropriate puzzles. • Encourage children to take apart and 'fix' appropriate toys, such as 'fixing' the wheels of a toy truck.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Shape and Space	
42-60+ months (3½ years to 5+ years)	• Follows directions and can use simple maps to find a location. • Identifies and names most shapes, both 2-D and 3-D, and knows the characteristics of different shapes. • Recognises geometry in their everyday world as well as the relationships between shapes, for instance, using two triangles to make a square or rectangle. • Completes increasingly complex puzzles. • Draws and constructs 2-D and 3-D models using a variety of materials. • Recognises the common features of regular and irregular shapes.	• Organise 'treasure hunts' and other games where children use simple maps to find things, and encourage children to make their own maps, e.g. of their street or preschool. • Continue to encourage children to build puzzles of increasing complexity. • Let children build with blocks, lego, empty boxes or milk cartons and combine, divide and change shapes. • Discuss shapes and how they are different ***("Boxes have corners and balls don't have corners"*** or ***"Triangles have three sides and diamonds have four sides"***) using examples in the everyday environment. • Look at children's shadows and talk about them and the ideas of two-dimensional and three-dimensional. • Point out irregular shapes (such as triangles with different size sides and angles) and discuss why they are still triangles.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Measurement	
Birth-22 months (birth to almost 2 years)	• Judges short distances and can adjust their reach to pick up an object. • Learns that different objects have different weights and changes their grasp accordingly. • Once they can walk, works out how to step or jump over a small object, and takes longer steps. • Explores quantity by filling and emptying containers with water or sand.	• Give babies ample opportunities to reach for and grasp a variety of objects. • Create opportunities for toddlers to jump over small objects or off a low step and play jumping games using the appropriate words such as 'high' and 'low'. • Encourage children to notice empty and full as they play with water or sand.
18-36 months (1½ years to 3 years)	• Thinks that a biscuit broken into pieces is more than the whole biscuit. • Understands words that describe different kinds of measurement, such as 'big' and 'small,' 'fast' and 'slow,' 'long' and 'short.' • When asked, can pick up objects in order of size (from biggest to smallest, for example). • Understands words related to time, such as 'now' and 'later.'	• Incorporate measurement words into everyday talk. • Give children opportunities to notice part and whole, break a biscuit in half and place it together again. • Play games noticing differences in size, such as looking for dogs with long or short tails. • Use stories to emphasise different sizes and quantities, such as 'Goldilocks and the Three Bears.'

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Measurement	
30-48 months **(2½ years to 4 years)**	• Understands and uses comparative terms like 'more,' 'less,' 'bigger' and 'slower.' • Accurately compares the sizes of two similar objects. • Uses informal ways of measuring, for example, fingers, pencils or a piece of string, to enable them to measure, estimate and compare. • Starts to use terms relating to time, such as 'tomorrow' and 'yesterday.' • Learns that some activities take longer than others.	• Measure together at home and make comparisons, e.g. measure the height of family members, using a hand to start with, or arrange the shoes in your family in a row from smallest to biggest. • Provide different tools for children to measure such as an egg-timer (time), balances (weight), rulers and pencils (length). • Create opportunities for children to measure quantities, e.g. by pouring water or sand from a jug into cups. • Explore weight with children, e.g. by comparing how heavy a bag of sugar and a loaf of bread are. • Encourage children to just take a guess about something – like who weighs the most – and then check it out for accuracy. • Talk with children about familiar routines and what happens at different times of day and night and create a simple chart together using pictures of the plans for the week ahead. • Explain terms such as 'yesterday' and 'tomorrow' by connecting them to events in the child's life, and use the names for different days and months.

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Measurement	
42-60+ months (3½ years to 5+ years)	• Recognises relationships between attributes, e.g. weight and size, size and volume. • Begins to use the vocabulary of standard units of measure. • Becomes aware of routines over longer periods and understands terminology relating to time, such as 'earlier,' 'later,' 'now,' 'today,' 'tomorrow.' • Becomes aware of the week, including the names of days and how the weekend is different, and changes in the seasons. • Starts to become familiar with the numbers on an analogue clock and knows that the short and long hands move round it.	• Pose problems and use activities to help children explore the relationship between different attributes and to discover that, for example, the size of an object is not directly related to its weight, e.g. an inflated balloon might be bigger than a small rock but it is lighter. • Cook together and measure how much you use of the different ingredients. • Organise your own 'Olympics' and measure how far the children can jump or how far they can throw a ball. • Compare one activity with another to work out what takes more time; use an egg-timer for measuring short periods of time. • Play games like 'Who can stand on one foot longer?' • Read and tell stories that involve sequencing events. • Encourage children to notice analogue clocks, explain the purpose of the short and long hands, and introduce time terms such as '12 o'clock' and 'half past three.'

	The child ***Learning highlights***	**The adult** ***Examples of ways to provide support***
	Data Collection	
Birth-22 months (birth to almost 2 years)	• Explores shapes, size, textures and weights of objects using all their senses.	• Make a collection of baby-safe objects from around the house in a 'treasure box' for babies to explore – e.g. a wooden spoon, saucepan, soft toy, plastic mug, sock and rattle.
18-36 months (1½ years to 3 years)	• Uses all senses to differentiate properties and functions of objects. • Collects and organises objects that share characteristics, such as hard and soft, red and blue. • Identifies their clothes or items that belong to them or other family members.	• Spend time in nature and take advantage of the endless opportunities for data collection, e.g. collecting leaves and comparing shape, texture and colour; or counting how many legs different animals have. • Let children help with sorting tasks, e.g. sort washing into piles together, looking for clothes that match, or sort toys into boxes together.
30-48 months (2½ years to 4 years)	• Sorts items into different categories; matches and makes comparisons to answer questions, e.g. which group has the most items. • Uses appropriate language to describe and compare different categories.	• Encourage and help children to organise their collections, by creating scrapbooks or special containers with dividers in (egg boxes can be used to start with). • Make available an interesting array of items with contrasting properties for children to sort and match, e.g. by colour, size, weight, texture, use or another characteristic. • Help children to develop rich descriptive language to compare and describe their collections.
42-60+ months (3½ years to 5+ years)	• Starts using written marks, symbols and objects to express mathematical ideas about groups and to create 'graphs.' • Can sort objects based on more than one characteristic. • Can draw conclusions from their information gathering, e.g. ***"My family's favourite drink is orange juice."*** • Can present their ideas and observations to others.	• Encourage children to sort objects in more sophisticated ways, based on more than one characteristic. • Encourage children to keep track of items they may find in nature or on outings, though drawings, symbols, markers or early writing. • Help children to create simple pictograms or graphs to represent their collections or findings.

Chapter 5

Strategies for success

THE PREVIOUS CHAPTERS HAVE set out both a framework for understanding mathematics development in the early years, and practical ideas and activities for delivering high quality teaching of early mathematics. However, for such teaching to become the norm, the policy framework and delivery context must both be appropriate. Policy in this sense refers to the laws and statutory and non-statutory guidelines that govern early years provision and ECD occupational qualifications. The delivery context refers to the enabling environment and resources, including evidenced learning programmes, which provide the practical tools for ECD practitioners, parents and community workers to ensure that all children are able to fulfil their potential.

A vital first step is to ensure that the early years curriculum framework in South Africa and the ECD occupational qualifications contain detailed and rigorous standards and guidance on how to support the development of early mathematics knowledge and skills in the years before a child starts school. The practical impact of this improved policy framework will, however, depend entirely on whether sufficient resources for implementation follow it. The quality of early mathematics learning will not improve consistently and for all children unless all early years settings are adequately supported and resourced. It is also essential that there is strong leadership nationally from a single lead department, which can be held to account for actual delivery and outcomes.

Early mathematics can be taught through simple and accessible activities but it does require a degree of specialised understanding and skills. ECD practitioners, home visitors and

community workers cannot therefore simply be encouraged to include early mathematics in their programmes – their success in ensuring that children enter Grade R with the necessary mathematics knowledge-base will depend on both the quality of training and the quality of teaching and learning resources available to them.

Initial and in-service training should ensure that all ECD practitioners are conversant with best practice around supporting early mathematics and trained in the teaching methods that have been shown to be most effective. Engaging and accessible learning programmes, with guidance on the different areas and stages of mathematics development alongside practical ideas and suggestions, enable ECD practitioners to structure teaching in a way that continually builds on prior knowledge. Simple materials – paper, pencils, rulers, blocks – can make a huge difference to practitioners' ability to successfully deliver early mathematics learning yet are frequently in short supply.

In many communities in South Africa, parents are inclined to minimise their role in their child's education by assuming that preschool and school are the 'proper' place for learning, and that they themselves lack the right knowledge to provide help and support. Yet parents and carers have the prime responsibility for creating rich mathematics learning environments in the months and years immediately after birth. Policies and programmes therefore need to address the twin challenges of **informing** and **empowering** parents in their role as a child's first teacher of language and mathematics. Parent classes and workshops can play an important part in motivating parents to support early mathematics development in the home, by equipping them with practical information, skills and confidence, and need to be much more widely available.

As individuals working directly alongside children during a crucial developmental period, home visitors and community, social and health workers all need to have better access to knowledge and resources about early mathematics development. These professionals often visit vulnerable families in their homes, and are well-placed to reinforce the importance of parental involvement in early mathematics learning and to model the practical strategies that can make any home a fertile learning environment. They, like ECD practitioners, need more specific guidance and practical ideas on supporting mathematics development in the early years, tailored to the home setting.

These various levels of delivery of a successful early mathematics strategy – government, ECD providers, community and home – can be brought together and capacitated through more effective sharing of knowledge and skills. Alliances and partnership and networking initiatives should be used to coordinate research, to capture and disperse learning and best practice, and to share information and resources.

Conclusion

There is extensive knowledge and evidence about how mathematics skills are developed from birth. We now need the policy framework, resources and resolve to use this knowledge to do things differently, in order to alter the academic trajectory of children from historically disadvantaged communities. This will require a committed and collaborative effort between all those involved in ECD service delivery – from policy-makers, ECD educators to crèche staff and parents.

Change and innovation at the point of delivery by individual providers is important, but it is essential that best practice and evidence-based approaches are much more widely understood and embraced. In South Africa, there are plenty of examples of good practice at programmatic level, delivered by state, non-profit and community providers. But these examples tend to be scattered, the exception rather than the norm, and until they are mainstreamed throughout ECD provision, many children will continue to lose out on quality learning and support. Furthermore, unless mathematics development is part of core programmes for ECD practitioners, home visitors and parents, interventions to support mathematics at school level will meet with only limited success among children who missed out on crucial development in their formative years.

In the meantime, small but significant advances in home learning environments, preschool provision and community programmes will continue to reap big rewards in terms of mathematics development and make a profound difference to the futures of young learners. We hope that this report will provide ECD trainers, providers and professionals, as well as parents and home visitors, with the guidance, practical ideas and motivation to be part of this transformative effort.

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